

Protsendid

osa = **TERVE** · osamäär

$$1\% = \frac{1}{100} = 0,01 \text{ osa tervest}$$

$$p\% = \frac{p}{100} \text{ osa tervest}$$

Osa leidmine tervest:

leiame $a\%$ arvust b $x = \frac{a}{100} \cdot b$

Terve leidmine osa järgi:

$a\%$ arvust on b $x = b : \frac{a}{100} = \frac{100b}{a}$

Mitu protsenti moodustab arv a arvust b ?

$$x = \frac{a}{b} \cdot 100\%$$

Muutumine protsentides a -st b -ni

$$x = \frac{|b-a|}{a} \cdot 100\%$$

Üks promill $1\text{‰} = \frac{1}{1000}$ osa tervest

Arvutamise abivalemid

$$a^2 - b^2 = (a-b)(a+b)$$

$$a-b = (\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})$$

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

$$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$$

$$(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$$

Trigonomeetria

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\frac{\sin \alpha}{\cos \alpha} = \tan \alpha$$

$$1 + \tan^2 \alpha = \frac{1}{\cos^2 \alpha}$$

$$\sin(-\alpha) = -\sin \alpha$$

$$\cos(-\alpha) = \cos \alpha$$

$$\tan(-\alpha) = -\tan \alpha$$

$$\sin(90^\circ - \alpha) = \cos \alpha$$

$$\cos(90^\circ - \alpha) = \sin \alpha$$

$$\tan(90^\circ - \alpha) = \frac{1}{\tan \alpha}$$

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

Astmed ja juured

$$a^0 = 1, a \neq 0 \quad a^1 = a$$

$$a^m \cdot a^n = a^{m+n} \quad a^m : a^n = a^{m-n} \quad (a \cdot b)^n = a^n \cdot b^n \quad (a : b)^n = a^n : b^n \quad (a^m)^n = a^{m \cdot n}$$

$$a^{-n} = \frac{1}{a^n} \quad a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

$$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b} \quad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \quad \sqrt[m]{\sqrt[n]{a^{m \cdot n}}} = \sqrt[n]{a^m} \quad \sqrt[n]{\sqrt[m]{a}} = \sqrt[n \cdot m]{a} \quad (\sqrt[n]{a})^m = \sqrt[n]{a^m}$$

$$\sqrt{a^2} = |a|$$

Ruutvõrrandi lahendamine

$$ax^2 + bx + c = 0$$

$$x^2 + px + q = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

Viete'i valemid $x_1 \cdot x_2 = q \quad x_1 + x_2 = -p$

Ruutkolmliikme tegurdamine $ax^2 + bx + c = a(x - x_1)(x - x_2)$

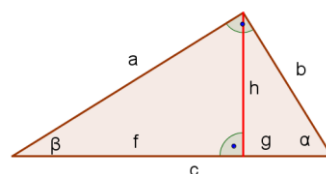
Täisnurkne kolmnurk

$$a^2 + b^2 = c^2 \quad \sin \alpha = \frac{a}{c} = \cos \beta$$

$$a^2 = f \cdot c \quad \cos \alpha = \frac{b}{c} = \sin \beta$$

$$b^2 = g \cdot c \quad \tan \alpha = \frac{a}{b} = \frac{1}{\tan \beta}$$

$$h^2 = f \cdot g \quad S = \frac{a \cdot b}{2} = \frac{c \cdot h}{2}$$



Kolmnurga lahendamine

Siinusteoreem $\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma} = 2R$

Koosinusteoreem $a^2 = b^2 + c^2 - 2bc \cos \alpha$

$$S = \frac{ah}{2} \quad S = \frac{ab \sin \gamma}{2} \quad S = \sqrt{p(p-a)(p-b)(p-c)} \quad S = \frac{abc}{4R} = pr$$

Vektorid ja sirged. Ringjoone võrrand

$$A(x_1; y_1), B(x_2; y_2) \quad \vec{AB} = (x_2 - x_1; y_2 - y_1) \quad |\vec{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\vec{u} = (x_1; y_1) \quad \vec{v} = (x_2; y_2) \quad \vec{u} \pm \vec{v} = (x_1 \pm x_2; y_1 \pm y_2) \quad k\vec{u} = (kx_1; ky_1)$$

$$\vec{u} \cdot \vec{v} = x_1x_2 + y_1y_2 \quad \vec{u} \cdot \vec{v} = |\vec{u}| |\vec{v}| \cos \varphi \quad \vec{u} \perp \vec{v} \Leftrightarrow \vec{u} \cdot \vec{v} = 0 \quad \vec{u} \parallel \vec{v} \Leftrightarrow \frac{x_1}{x_2} = \frac{y_1}{y_2}$$

Tõusu ja algordinaadiga määratud sirge $y = kx + b$

Punkti ja tõusuga määratud sirge $y - y_1 = k(x - x_1)$

Kahe punktiga määratud sirge $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$

Punkti ja vektoriga määratud sirge $\frac{x - x_1}{s_x} = \frac{y - y_1}{s_y}$

Sirge üldvõrrand $Ax + By = C, \quad \vec{s} = (-B; A)$

Ristivate sirgete tõusude korrutis: $k_1 \cdot k_2 = -1$

Ringjoone võrrand

$K(0;0)$, raadius r $x^2 + y^2 = r^2$ $K(a;b)$, raadius r $(x-a)^2 + (y-b)^2 = r^2$

Trigonomeetriliste funktsioonide väärtused

 α $\sin \alpha$ $\cos \alpha$ $\tan \alpha$
 0° 0 1 0

 30° 0,5 $\frac{\sqrt{3}}{2}$ $\frac{\sqrt{3}}{3}$
 45° $\frac{\sqrt{2}}{2}$ $\frac{\sqrt{2}}{2}$ 1

 60° $\frac{\sqrt{3}}{2}$ 0,5 $\sqrt{3}$
 90° 1 0 -

Trigonomeetrilised võrrandid

$$\sin x = m \quad x = (-1)^n \arcsin m + n\pi$$

$$\cos x = m \quad x = \pm \arccos m + 2n\pi$$

$$\tan x = m \quad x = \arctan m + n\pi$$

Aritmeetiline jada

$$a_n = a_1 + (n-1)d \quad a_k = \frac{a_{k-1} + a_{k+1}}{2}$$

$$S_n = \frac{a_1 + a_n}{2} \cdot n = \frac{2a_1 + (n-1)d}{2} \cdot n$$

Geomeetriline jada

$$a_n = a_1 \cdot q^{n-1} \quad a_k = \sqrt{a_{k-1} \cdot a_{k+1}}$$

$$S_n = \frac{a_1(q^n - 1)}{q - 1}$$

$$\text{Hääbuva jada summa } S = \frac{a_1}{1 - q}$$

Eksponentsiaalne kasvamine (kahanemine)

$$L = A \left(1 \pm \frac{p}{100} \right)^n$$

L – lõppväärtus, A – algväärtus,

p – muutus protsentides,

n – tsüklite arv

Funktsiooni tuletis

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$u = u(x), \quad v = v(x)$$

$$(u \pm v)' = u' \pm v' \quad (u \cdot v)' = u'v + v'u$$

$$\left(\frac{u}{v} \right)' = \frac{u'v - v'u}{v^2} \quad (c \cdot u)' = c \cdot u'$$

$$c' = 0 \quad x' = 1 \quad (x^n)' = n \cdot x^{n-1} \quad (\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

$$\left(\frac{1}{x} \right)' = -\frac{1}{x^2} \quad (e^x)' = e^x \quad (\ln x)' = \frac{1}{x}$$

$$(\log_a x)' = \frac{1}{x \cdot \ln a}$$

$$(\sin x)' = \cos x \quad (\cos x)' = -\sin x$$

$$(\tan x)' = \frac{1}{\cos^2 x}$$

Logaritm

$$\log_a b = c \Leftrightarrow a^c = b, \quad \log_a 1 = 0, \quad \log_a a = 1, \quad \log_{10} x = \log x, \quad \log_e a = \ln a$$

$$\log_a (x \cdot y) = \log_a x + \log_a y, \quad \log_a \frac{x}{y} = \log_a x - \log_a y, \quad \log_a x^n = n \cdot \log_a x$$

$$\log_a b = \frac{1}{\log_b a}, \quad \log_a b = \frac{\log_c b}{\log_c a}, \quad \log 10 = 1, \quad \ln e = 1$$

Funktsiooni uurimine

Nullkohad: lahendatakse võrrand $f(x) = 0$

Positiivsus (negatiivsus): lahendatakse võrratus $f(x) > 0, f(x) < 0$

Ekstreemumkoha tarvilik tingimus: $f'(x) = 0$

Kasvamisvahemikud: $f'(x) > 0$, **kahanemisvahemikud:** $f'(x) < 0$

Puutuja kohal x_0 : $y - y_0 = f'(x_0)(x - x_0)$

Tõenäosus

$$p = \frac{m}{n} \quad n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-1) \cdot n \quad C_n^m = \frac{n!}{m!(n-m)!} \quad A_n^m = \frac{n!}{(n-m)!}$$

$$P_{n,k} = C_n^k p^k q^{n-k}$$

Integraal

$$\int dx = x + C \quad \int x^n dx = \frac{x^{n+1}}{n+1} + C \quad \int e^x dx = e^x + C \quad \int \frac{1}{x} dx = \ln|x| + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C \quad \int \sin x dx = -\cos x + C \quad \int \cos x dx = \sin x + C$$

$$\text{Newton-Leibnizi valem: } \int_a^b f(x) dx = F(b) - F(a)$$

Tasandi- ja ruumikujundid

Ruut $S = a^2$ $P = 4a$

Ristkülik $S = ab$ $P = 2(a + b)$

$$S = ah = ab \sin \gamma$$

Rööpkülik $P = 2(a + b)$ $d_1^2 + d_2^2 = 2(a^2 + b^2)$

Romb $S = \frac{d_1 \cdot d_2}{2} = ah$ $P = 4a$

Trapets $k = \frac{a+b}{2}$ $S = k \cdot h = \frac{a+b}{2} \cdot h$

Võrdkülgne kolmnurk $P = 3a$ $S = \frac{a^2 \sqrt{3}}{4}$

Ringjoon ja ring $C = 2\pi r$ $S = \pi r^2$

Kaare pikkus, sektori pindala $L = \frac{\pi r \alpha^\circ}{180^\circ}$ $S = \frac{\pi r^2 \alpha^\circ}{360^\circ}$

Kuup $S = 6a^2$ $V = a^3$

Risttahukas $S = 2(ab + ac + bc)$ $V = abc$

Püströöptahukas $S_t = 2S_p + S_k$ $S_k = P \cdot H$ $V = S_p \cdot H$

Silinder $S_p = \pi r^2$ $S_k = 2\pi rh$ $S_t = 2S_p + S_k$ $V = \pi r^2 h$

Koonus $S_p = \pi r^2$ $S_k = \pi r m$ $S_t = S_p + S_k$ $V = \frac{1}{3} \pi r^2 h$

Kera $S = 4\pi R^2$ $V = \frac{4}{3} \pi R^3$

Regulaarne tetraeder $S_t = 4 \cdot \frac{a^2 \sqrt{3}}{4} = a^2 \sqrt{3}$ $V = \frac{a^3 \sqrt{2}}{12}$

Püramiidi ruumala $V = \frac{1}{3} S_p \cdot H$